THE TEMPERATURE FIELD ARISING WITH THE MOTION OF A REACTING SPHERE WITH SMALL FINITE PÉCLET AND REYNOLDS NUMBERS

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A solution is obtained to the problem of the temperature field induced by a surface chemical reaction outside and inside a spherical particle moving in a liquid at small finite Reynolds numbers. The problem is solved by the method of coalescing asymptotic expansions [1] in terms of a small thermal Péclet number, under the assumption that the reaction takes place under diffusion conditions. There has been obtained previously the concentration field of a reagent [2] around a moving particle on whose surface a chemical reaction of the first order is taking place, with small finite Reynolds and Péclet numbers. Below, using the results of [2], there is determined the temperature field outside and inside of a reacting sphere, induced by the thermal effect of the reaction.

1. Statement of Problem. Method of Solution. We consider a spherical particle moving in a viscous liquid; on the surface of the particle there takes place a chemical reaction, accompanied by the evolution (absorption) of heat. At a large distance from the particle the flow of the liquid is assumed to be homogeneous, so that the velocity of the flow, the concentration of the reacting substance, and the temperature of the flow have the constant values U, c_0, and T_0, respectively. In the neighborhood of the particle, due to the perturbations which it introduces, the homogeneity of the velocity, concentration, and temperature fields breaks down. The distribution of the velocities at small finite Reynolds numbers was found in [3, 4], and the distribution of the concentrations at small finite Reynolds and Péclet numbers in [2]. Based on the results of [2], under the same conditions we shall find the distributions of the temperature outside and inside of a particle in the case of small finite values of the thermal Péclet number.

The thermal conductivity equations, in dimensionless variables, for the regions outside and inside a sphere can be written in the form

$$\Delta\varphi = \frac{P_\chi}{r^2}\frac{\partial(\psi, \varphi)}{\partial(r, \mu)}, \quad 1 \leqslant r < \infty \qquad (\mu = \cos\theta) \tag{1.1}$$

$$\Delta\Phi = 0, \qquad 0 \leqslant r \leqslant 1$$

$$\left(\varphi = Lc_p\frac{T - T_0}{h}, \quad \Phi = Lc_p\frac{T_1 - T_0}{h}, \quad L = \frac{\chi}{D}, \quad P_\chi = \frac{Ua}{\chi}\right) \tag{1.2}$$

Here $T(r, \mu)$ and $T_1(r, \mu)$ are the temperature distributions in the flow and inside the particle; r is a radial coordinate, referred to the radius of the particle a; θ is the angle between the radius vector and the direction of the velocity of the unperturbed flow; Δ is an axisymmetric spherical Laplace operator; c_p is the heat capacity of the liquid; h is the heat of reaction; χ is the thermal diffusivity coefficient; D is the diffusion coefficient; L is the Lewis number; P_χ is the thermal Péclet number; ψ is the dimensionless (referred to Ua^2) flow function.

For the dimensionless flow function we use the internal and external asymptotic expansions obtained in [3, 4]; these are written in the form

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$$\psi_* = \frac{1}{4}(r-1)^2(1-\mu^2)\left[\left(1+\frac{3}{8\sigma}P_\chi+\frac{9}{40\sigma^2}P_\chi^2\ln P_\chi\right)\left(2+\frac{1}{r}\right)-\right.$$
$$\left.-\frac{3}{8\sigma}P_\chi\left(2+\frac{1}{r}+\frac{1}{r^2}\right)\mu\right]+O(P_\chi^2) \quad \left(\sigma=\frac{\nu}{\chi}\right) \tag{1.3}$$

$$\psi^* = \frac{1}{2}\rho^2(1-\mu^2)-\frac{3}{2}\sigma P_\chi(1+\mu)\left[1-\exp\left(-\rho\frac{1-\mu}{2\sigma}\right)\right]+O(P_\chi^2) \tag{1.4}$$
$$(\psi^*=\psi P_\chi^2, \quad \rho=rP_\chi)$$

Here and in what follows, an asterisk below or above denotes internal or external asymptotic expansion, respectively; ρ is the compressed radial coordinate; σ is the Prandtl number.

The boundary conditions expressing the homogeneity of the temperature far from the particle, the continuity of the temperature and the heat balance at its surface, as well as the boundedness of the temperature at the center of the particle have the form

$$r\to\infty, \quad \varphi\to 0 \tag{1.5}$$
$$r=1, \quad \varphi=\Phi \tag{1.6}$$
$$r=1, \quad \frac{\partial\varphi}{\partial r}-\delta\frac{\partial\Phi}{\partial r}=\frac{\partial\xi}{\partial r} \quad \left(\delta=\frac{\lambda_1}{\lambda}, \quad \xi=\frac{c_0-c}{c_0}\right) \tag{1.7}$$
$$r=0, \quad \Phi<\infty \tag{1.8}$$

Here δ is the ratio of the coefficients of thermal conductivity of the particle λ_1 and the liquid λ; ξ is the degree of progress of the reaction; c is the concentration of the reagent in the flow.

The boundary condition (1.7) contains the value of the normal component of the gradient of the progress of the reaction at the surface of the particle. To determine this value, we use the results of [2], limiting ourselves to the case of a reaction taking place in the diffusion region (the reaction rate constant is large in comparison with the ratio of the diffusion coefficient to the radius of the particle). We obtain

$$\left(\frac{\partial\xi}{\partial r}\right)_{r=1} = -1-\left(\frac{1}{2}-\frac{3}{8}\mu\right)LP_\chi-\frac{1}{2}L^2P_\chi^2\ln P_\chi -$$
$$-\left[\frac{1}{2}\left(Q+\ln L\right)+\frac{9}{16}\left(1-\frac{1}{4S}\right)\mu-\frac{1}{64}\left(\frac{33}{7}-\frac{13}{5S}\right)\frac{3\mu^2-1}{2}\right]L^2P_\chi^2 -$$
$$-\frac{1}{4}\left[1-\frac{3}{4}\left(1+\frac{9}{20S^2}\right)\mu\right]L^3P_\chi^3\ln P_\chi+O(P_\chi^3) \tag{1.9}$$
$$Q=-\frac{173}{160}+\gamma+\frac{S^2}{2}-\frac{S}{4}-(S+1)^2\left(\frac{S}{2}-1\right)\ln\left(1+\frac{1}{S}\right) \quad (S=\sigma L)$$

The above formulated problem (1.1)-(1.9) is close to that considered in [2]. To solve it, as in [2], we use the method of coalescing asymptotic expansions.

The internal and external asymptotic expansions of the reduced temperature outside of the sphere will be sought in the form

$$\varphi_* = \sum_{n=0}^{\infty}\alpha_n(P_\chi)\varphi_n(r,\mu) \tag{1.10}$$

$$\varphi^* = \sum_{n=0}^{\infty}\alpha^{(n)}(P_\chi)\varphi^{(n)}(\rho,\mu) \tag{1.11}$$

The asymptotic expansion of the solution inside the sphere, as is indicated by the conditions at the surface (1.6) and (1.7), must be sought in a form analogous to (1.10):

$$\Phi = \sum_{n=0}^{\infty}\alpha_n(P_\chi)\Phi_n(r,\mu) \tag{1.12}$$

With respect to the functions $\alpha_n(P_\chi)$ and $\alpha^{(n)}(P_\chi)$ it is assumed only that the order of their smallness with respect to P_χ increases with an increase in the value of n.

The terms of the expansions (1.10), (1.12) will be determined consecutively from Eqs. (1.1), (1.2) with the boundary conditions (1.6)-(1.8), taking account of (1.9); the flow function entering into (1.1) is given by the internal expansion (1.3).

We determine the terms of the expansion (1.11) from Eq. (1.1) and condition (1.5), written in external variables ($\rho = rP_\chi$, $\psi^* = \psi P_\chi^2$):

$$\Delta^*\varphi^* = \frac{1}{\rho^2}\frac{\partial(\psi^*, \varphi^*)}{\partial(\rho, \mu)}; \quad \rho \to \infty, \quad \varphi^* \to 0 \tag{1.13}$$

Here Δ^* is an axisymmetric Laplace operator, obtained from Δ by the replacement of r by ρ; the function $\psi^* = \psi^*(\rho, \mu)$ is determined by the expansion (1.4).

The arbitrary constants, which remain indeterminate in each stage of the solution of the problems (1.1)-(1.3), (1.6)-(1.9), and (1.13), (1.4), are determined by coalescence of the expansions (1.10) and (1.11).

2. Zero and First Approximations. The construction of the solution starts with determination of the zero term of the external expansion. Obviously, the problem (1.13), (1.4) is satisfied by the solution

$$\varphi^{(0)} = 0 \tag{2.1}$$

For the zero terms of the expansions (1.10), (1.12), from (1.1)-(1.3), (1.6)-(1.9) with $P_\chi = 0$ we obtain

$$\begin{gathered}\varphi_0 = \sum_{m=0}^{\infty}(a_{0m}r^m + b_{0m}r^{-m-1})P_m(\mu)\\ \Phi_0 = \sum_{m=0}^{\infty}(a_{0m} + b_{0m})r^mP_m(\mu)\\ b_{00} = 1, \quad (\delta - 1)ma_{0m} + (\delta m + m + 1)b_{0m} = 0 \qquad (m \geqslant 1)\end{gathered} \tag{2.2}$$

Here $P_m(\mu)$ are Legendre polynomials.

Determining the constants a_{0m} and b_{0m} by the asymptotic coalescence of φ_0 with $\varphi^{(0)}$, we obtain

$$\varphi_0 = r^{-1}, \qquad \Phi_0 = 1 \tag{2.3}$$

It is obvious that expressions (2.3) describe the temperature distributions outside and inside of a stationary spherical particle with a surface nonisothermal reaction taking place in the diffusion region.

Writing φ_0 as a function of the external variable ρ, we find that in the external expansion, $\alpha^{(1)} = P_\chi$. Then, to determine the first approximation $\varphi^{(1)}$ for the external expansion we obtain from (1.13), (1.4) a problem identical to that considered earlier [2] in finding $\xi^{(1)}$. With the coalescence of $\varphi^{(1)}$ and φ_0 it is evident from (2.3) that φ_0 differs from the corresponding function ξ_0 from [2] only by a constant with r^{-1}. Therefore, we can immediately write $\varphi^{(1)}$ in the form

$$\varphi^{(1)} = \rho^{-1}\exp\left[{}^1/_2\rho(\mu - 1)\right] \tag{2.4}$$

Going over in (2.4) to the internal variable r and expanding $\varphi^{(1)}$ in a series in terms of P_χ, we find that in the external expansion, $\alpha_1 = P_\chi$. To determine φ_1 and Φ_1, in a two-term internal expansion we obtain from (1.1)-(1.3), (1.6)-(1.9) the following problem:

$$\begin{gathered}\Delta\varphi_1 = -\frac{1}{r^2}\left(1 - \frac{3}{2r} + \frac{1}{2r^3}\right)\mu, \quad \Delta\Phi_1 = 0\\ r = 1, \quad \varphi_1 = \Phi_1, \quad \frac{\partial\varphi_1}{\partial r} - \delta\frac{\partial\Phi_1}{\partial r} = -L\left(\frac{1}{2} - \frac{3}{8}\mu\right)\\ r = 0, \quad \Phi_1 < \infty\end{gathered} \tag{2.5}$$

The solution of problem (2.5) has the form

$$\begin{gathered}\varphi_1 = \left(\frac{1}{2} - \frac{3}{4r} - \frac{1}{8r^3}\right)\mu + \sum_{m=0}^{\infty}(a_{1m}r^m + b_{1m}r^{-m-1})P_m(\mu)\\ \Phi_1 = -\frac{3}{8}r\mu + \sum_{m=0}^{\infty}(a_{1m} + b_{1m})r^mP_m(\mu)\\ b_{10} = {}^1/_2L, \qquad (\delta - 1)a_{11} + (\delta + 2)b_{11} = {}^3/_8(3 + \delta - L)\\ (\delta - 1)ma_{1m} + (\delta m + m + 1)b_{1m} = 0 \qquad (m \geqslant 2)\end{gathered} \tag{2.6}$$

By coalescing $\varphi_{*1} = \varphi_0 + P_\chi \varphi_1$ with $\varphi^{*(1)} = \varphi^{(0)} + P_\chi \varphi^{(1)}$, we determine the constants a_{1m} and b_{1m}. We obtain

$$\varphi_1 = -\frac{1}{2} + \frac{L}{2r} + \left(\frac{1}{2} - \frac{3}{4r} + \frac{3}{8r^2}\frac{3+\delta-L}{2+\delta} - \frac{1}{8r^3}\right)\mu \tag{2.7}$$

$$\Phi_1 = (L-1)\left(\frac{1}{2} - \frac{3}{8}\frac{r}{2+\delta}\mu\right) \tag{2.8}$$

3. Third and Fourth Approximations. The two-term internal expansion, written in external variables, shows that the three-term external expansion has the form

$$\varphi^{*(2)} = \varphi^{(0)} + P_\chi \varphi^{(1)} + P_\chi^2 \varphi^{(2)} \tag{3.1}$$

For $\varphi^{(2)}$, from (1.13), using (1.4), (2.1), (2.4), we obtain

$$\Lambda\varphi^{(2)} = \frac{3}{4}\frac{\sigma}{\rho^3}\left(\frac{\sigma+1}{\sigma} + \frac{2}{\rho} - \frac{\sigma-1}{\sigma}\mu\right)\exp\left(\rho\frac{\sigma+1}{\sigma}\frac{\mu-1}{2}\right) -$$
$$-\frac{3}{4}\frac{\sigma}{\rho^3}\left(1 + \frac{2}{\rho} - \mu\right)\exp\left(\rho\frac{\mu-1}{2}\right), \qquad \Lambda = \Delta^* - \mu\frac{\partial}{\partial\rho} - \frac{1-\mu^2}{\rho}\frac{\partial}{\partial\mu} \tag{3.2}$$
$$\rho \to \infty, \qquad \varphi^{(2)} \to 0$$

The problem (3.2) coincides with that considered previously [2]. Using the solution obtained in [2], and determining the unknown constants by coalescing $\varphi^{*(2)}(\rho, \mu)$ and $\varphi_{*1}(r, \mu)$, we find an expression for the asymptotic curve $\varphi^{(2)}(\rho, \mu)$:

$$\varphi^{(2)}(\rho, \mu) = \frac{1}{2\rho}\left(L - \frac{3}{2}\mu\right) - \frac{1}{2}\ln\rho + \zeta(L, \sigma) + \tag{3.3}$$
$$+\frac{1}{4}\left[L + \frac{3}{4\sigma} - \left(\frac{3}{2} + \frac{1}{5\sigma} - \frac{1}{10\sigma^2}\right)\rho\ln\rho\right]\mu - \frac{5}{24}\left(1 + \frac{3}{10\sigma}\right)\frac{3\mu^2-1}{2} + O(\rho)$$
$$\zeta(L, \sigma) = -\frac{1}{4}\sigma^2 + \frac{1}{8}\sigma + \frac{25}{24} - \frac{L}{4} - \frac{\gamma}{2} + \frac{1}{4}(\sigma+1)^2(\sigma-2)\ln\left(1 + \frac{1}{\sigma}\right)$$

Formula (3.3) shows, in particular, that there is a logarithmic singularity in the second approximation for the external expansion. Going over in (3.3) to an internal variable, we find the coefficient α_2 in the internal expansion (1.10):

$$\alpha_2(P_\chi) = P_\chi^2 \ln P_\chi$$

From (1.10), (1.12), (1.1)-(1.3), (1.6)-(1.9), for φ_2 and Φ_2 relationships analogous to relationships (2.2) (with the replacements $a_{0m} \to a_{2m}$, $b_{0m} \to b_{2m}$), with the sole difference that $b_{02} = \frac{1}{2}L^2$, are obtained. Therefore, after coalescence of φ_{*2} and $\varphi^{*(2)}$, we immediately obtain

$$\varphi_2 = \tfrac{1}{2}(L^2/r - 1), \qquad \Phi_2 = \tfrac{1}{2}(L^2 - 1)$$

Simultaneously with the second approximation, relationship (3.3) makes it possible to obtain also the third approximation for the internal expansion. It follows from (3.3) that $\alpha_3 = P_\chi^2$. The function $\varphi_3(r, \mu)$ satisfies the equation

$$\Delta\varphi_3 = \sum_{n=0}^{2} Y_n(r) P_n(\mu) \tag{3.4}$$
$$Y_0(r) = \frac{1}{3r} - \frac{1}{2r^2} + \frac{3}{16}\left(\eta - \frac{2}{9}\right)\frac{1}{r^4} + \frac{1}{8r^5} - \frac{3\eta}{16r^6} + \frac{1}{12r^7}$$
$$Y_1(r) = \left(\frac{L}{4} + \frac{3}{16\sigma}\right)\left(-\frac{2}{r^2} + \frac{3}{r^3} - \frac{1}{r^5}\right), \qquad \eta = \frac{3+\delta-L}{2+\delta}$$
$$Y_2 = -\frac{1}{3r} + \frac{5}{4r^2} - \frac{3}{4}\left(\eta + \frac{3}{2}\right)\frac{1}{r^3} + \frac{15}{16}\left(\eta + \frac{4}{9}\right)\frac{1}{r^4} - \frac{5}{16r^5} -$$
$$-\frac{3\eta}{16r^6} + \frac{5}{48r^7} + \frac{3}{16\sigma}\left(\frac{2}{r^2} - \frac{3}{r^3} + \frac{1}{r^4} - \frac{1}{r^5} + \frac{1}{r^6}\right)$$

The general solution of Eq. (3.4) is

$$\varphi_3 = \sum_{m=0}^{\infty} [\varphi_{3,m}(r) + a_{3m} r^m + b_{3m} r^{-m-1}] P_m(\mu)$$

$$\varphi_{3,0} = \frac{r}{6} - \frac{\ln r}{2} + \frac{3}{32}\left(\eta - \frac{2}{9}\right)\frac{1}{r^2} + \frac{1}{48r^3} - \frac{\eta}{64r^4} + \frac{1}{240r^5}$$

$$\varphi_{3,1} = \left(\frac{L}{4} + \frac{3}{16\sigma}\right)\left(1 - \frac{3}{2r} - \frac{1}{4r^3}\right) \tag{3.5}$$

$$\varphi_{3,2} = \frac{r}{12} - \frac{5}{24} + \frac{1}{8}\left(\eta + \frac{3}{2}\right)\frac{1}{r} - \frac{15}{64}\left(\eta + \frac{4}{9}\right)\frac{1}{r^2} +$$

$$+ \frac{\ln r}{16r^3} - \frac{\eta}{32r^4} + \frac{5}{672r^5} + \frac{3}{16\sigma}\left(-\frac{1}{3} + \frac{1}{2r} - \frac{1}{4r^2} + \frac{\ln r}{5r^3} + \frac{1}{6r^4}\right)$$

$$\varphi_{3,m} = 0 \qquad (m \geqslant 3)$$

Coalescing $\varphi_{*3} = \varphi_0 + P_\chi \varphi_1 + P_\chi^2 \ln P_\chi \varphi_2 + P_\chi^2 \varphi_3$ for $r \to \infty$ and the external expansion $\varphi^{*(2)} = P_\chi \varphi^{(1)} + P_\chi^2 \varphi^{(2)}$ at $\rho \to 0$, we find the constants a_{3m}:

$$a_{30} = \zeta(L, \sigma), \quad a_{31} = -{}^1\!/_4, \quad a_{3m} = 0 \quad (m \geqslant 2) \tag{3.6}$$

From (1.2), (1.12), taking account of (1.6), (1.8), for a term of the order P_χ^2 in the asymptotic expansion of the solution within the sphere we have

$$\Phi_3 = \sum_{m=0}^{\infty} [\varphi_{3,m}(1) + a_{3m} + b_{3m}] r^m P_m(\mu) \tag{3.7}$$

Boundary condition (1.7) now permits determining the constants b_{3m}. By virtue of (3.5)-(3.7), (1.9) we obtain the relationships

$$\varphi_{30}'(1) - b_{30} = -\frac{L^2}{2}(Q + \ln L)$$

$$\varphi_{31}'(1) + \frac{\delta - 1}{4} + (2 + \delta) b_{31} - \delta \varphi_{31}(1) = \frac{9L^2}{16}\left(\frac{1}{4\sigma L} - 1\right)$$

$$\varphi_{32}'(1) - (3 + 2\delta) b_{32} - 2\delta \varphi_{32}(1) = \frac{L^2}{64}\left(\frac{33}{7} - \frac{13}{56\sigma L}\right)$$

$$b_{3m} = 0 \quad (m \geqslant 3)$$

Hence

$$b_{30} = -\frac{\eta + 3}{8} + \frac{L^2}{2}(Q + \ln L)$$

$$b_{31} = \frac{9\eta}{64\sigma} + \left(1 - \frac{3}{4}\eta\right)\left(1 + \frac{3}{4}L\right) + \frac{9L^2}{16}\frac{1 + \delta}{2 + \delta}$$

$$b_{32} = \frac{1}{3 + 2\delta}\left(\frac{87 + 46\delta}{672} + \frac{15 + 9\delta}{32}\eta - \frac{2 + \delta}{32\sigma}\eta + \frac{2 + 3L}{320\sigma} - \frac{33L^2}{448}\right) \tag{3.8}$$

Thus, the third term of the internal expansion has the form (3.5), where the coefficients a_{3m}, b_{3m} are given by the formulas (3.6), (3.8), respectively.

4. <u>The Temperature Field Outside and Inside of a Particle. Heat Flux at the Surface of a Particle.</u> The expression for the temperature distribution in the flow surrounding the particle has the form (near the particle)

$$\varphi_* = \frac{1}{r} + P_\chi\left[-\frac{1}{2} + \frac{L}{2r} + \left(\frac{1}{2} - \frac{3}{4r} + \frac{3\eta}{8r^2} - \frac{1}{8r^3}\right)\mu\right] + \tag{4.1}$$

$$+ P_\chi^2 \ln P_\chi\left(-\frac{1}{2} + \frac{L^2}{2r}\right) + P_\chi^2\left\{\zeta - \frac{r}{4} + \sum_{m=0}^{2} [\varphi_{3,m}(r) + b_{3m} r^{-m-1}] P_m(\mu)\right\}$$

Here $\varphi_{3,m}(r)$ and b_{3m} are determined using formulas (3.5) and (3.8).

The temperature field inside the particle is

$$\Phi = 1 + P_\chi (L - 1)\left(\frac{1}{2} - \frac{3}{8}\frac{r\mu}{2 + \delta}\right) + \frac{1}{2} P_\chi^2 \ln P_\chi (L^2 - 1) + P_\chi^2 \sum_{m=0}^{2} A_{3m} r^m P_m(\mu) \tag{4.2}$$

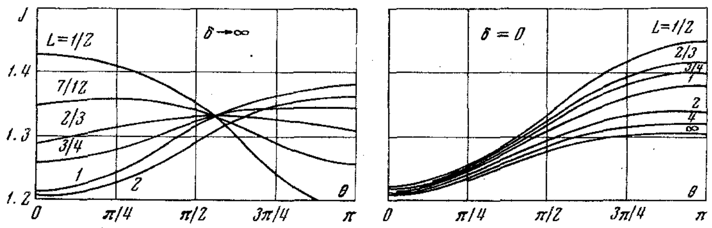

Fig. 1

$$A_{30} = -\frac{49}{240} - \frac{3\eta}{64} + \zeta + \frac{L^2}{2}(Q + \ln L)$$

$$A_{31} = \frac{1-L}{64\sigma(2+\delta)}(9 - 12\sigma L - 16\sigma), \quad A_{32} = \frac{1-L}{64(3+2\delta)}\left[\frac{33}{7}(L+1) + \frac{3}{2+\delta} - \frac{13}{5\sigma}\right]$$

With L = 1, as is easily verified, $\Phi = 1$. This confirms the fact that in the case of similarity of the concentration and temperature fields the temperature of the particle should remain constant.

The thermal Nusselt number is determined by the formula

$$N = -\int_{-1}^{1} \frac{\partial\varphi}{\partial r}\Big|_{r=1} d\mu \tag{4.3}$$

A calculation shows that the Nusselt number can be written in the following form, coinciding with the corresponding expression of [2], with

$$k \to \infty : N = 2 + P + P^2 \ln P + P^2 Q$$

where Q is determined by (1.9); $P = P_\chi L$.

Figure 1 shows the distribution of the local heat flux on a spherical particle as a function of the Lewis number. The local heat flux on the particle is equal to

$$\begin{aligned} j = -\frac{\partial\varphi}{\partial r}\Big|_{r=1} &= 1 + P_\chi\left[\frac{L}{2} - \left(\frac{9}{8} - \frac{3}{4}\,\frac{3+\delta-L}{2+\delta}\right)\mu\right] + \frac{L^2}{2} P_\chi^2 \ln P_\chi + \\ &+ P_\chi^2\left\{\frac{L^2}{2}(Q+\ln L) - \frac{3}{4(2+\delta)}\left[\delta\left(\frac{L}{4} + \frac{3}{16\sigma} - 1\right) + \frac{3L}{8\sigma} - \frac{3L^2}{2}\right]\mu - \right. \\ &\left. - \frac{3}{4(3+2\delta)}\left[\frac{29+11\delta-7L}{56(2+\delta)}\delta - \frac{13\delta}{120\sigma} + \frac{L^2}{16}\left(\frac{33}{7} - \frac{13}{5\sigma L}\right)\right]\frac{3\mu^2-1}{2}\right\} \end{aligned}$$

In addition to the Lewis number, there enter here as parameters also the Prandtl number $\sigma = \nu/\chi$, the Péclet number, and the ratio of the thermal conductivities of the sphere and the surrounding medium. The values of σ and of the diffusional Péclet number are fixed; $\sigma = 1$, $P = 1/2$. The L numbers are taken within the limits from 0.5 to 2, which corresponds to the real values for many gases. Then

$$P_\chi = P / L = {}^1/_2 L^{-1}$$

$$j = 1.163 + 0.125Q - P_\chi 0.375\mu - P_\chi^2\left[({}^3/_{16}L - {}^{39}/_4)\mu + 0.034(3\mu^2 - 1)/2\right] \quad \text{at} \quad \delta \to \infty$$

$$j = 1.163 + 0.125Q - 0.1875\mu - 0.25\left[({}^9/_{64}L^{-1} - {}^9/_{16})\mu + {}^3/_{128}({}^{33}/_7 - {}^{13}/_5 L^{-1})(3\mu^2 - 1)/2\right] \quad \text{at} \quad \delta = 0$$

The curves show that, with $\delta \to \infty$, at the point where the flow encounters the particle ($\theta = \pi$) the heat flux at first rises with an increase in the Lewis number, then, when the Lewis number becomes greater than unity, starts to fall slowly. This is evident also from the expression for j with $\delta \to \infty$: $\partial j/\partial L > 0$ at L = 1; $\partial j/\partial L < 0$ at L = 2.

In the front part there exists a value of the angle $\theta \approx 103°$ at which j is practically independent of the Lewis number for all values of $L \le 1$. At the rear point ($\theta = 0$) the density of the heat flux falls with an increase in L.

In the case $\delta = 0$ the local heat flux to the particle decreases with an increase in L, and for each individual value of L it decreases smoothly from its greatest value at the point of impact of the flow ($\theta = \pi$) to its lowest value at the rear point ($\theta = 0$).

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